TRAJECTORY OPTIMIZATION FOR DEFLECTION OF ASTEROID 2024 PDC25 USING GENETIC ALGORITHMS AND DEPARTURE VIA LUNAR SWING-BYS Rebeca de Souza Ribeiro[1], Guilherme Marcos Neves[1], [1]National Institute for Space Research, Av. dos Astronautas, 1758, Sao Jose dos Campos, Sao Paulo, 12227-010, Brazil; rebeca.ribeiro@inpe.br, guilherme.marcos@inpe.br

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Introduction: On 24 December 2024, the scientific community turned its attention to the sky, due to the discovery of the asteroid "2024 YR4" [1]. The discovery of the Apollo-type asteroid triggered the first response layer of planetary defense, with further observations of the object being prompted to investigate its impact probability with the Earth. Investigating possible threats to our planet by Near-Earth Objects (NEOs) is an ongoing and crucial task for maintaining the well-being of Earth. To raise awareness of the potential hazard that NEOs might pose to life on Earth, the 2025 Planetary Defense Conference proposes a hypothetical asteroid impact scenario with the discovery of the asteroid "2024 PDC25".

To address this exercise, this study investigates the planning of an impact mission to the asteroid. It considers different departure trajectories from the Earth-Moon system combined with genetic algorithms to analyze the heliocentric transfer. In other words, starting from a Low-Earth Orbit (LEO), an increment of velocity (ΔV) is applied to a spacecraft parked in this orbit to insert it into:

- A hyperbolic escape trajectory, in a similar approach to the patched conics approximation, or
- A Trajectory G of Escape (TGE), a trajectory originated from a periodic orbit around the Lagrangian point L_1 in the Earth-Moon system, in which a lunar swing-by is performed.

After the departure of the Earth-Moon system, via either of these trajectories, a Genetic Algorithm (GA) is utilized considering a single-impulsive maneuver between the Earth and 2024 PDC25 and uses the solution of Lambert's Problem in the fitness function.

Different launch dates are selected to investigate the optimal transfer, in terms of the ΔV requirements to complete the transfer, and to analyze how efficient earlier impacts might be in the deviation of the asteroid away from the Earth, also considering that the lunar gravity assist adds time to the duration of the mission.

a) Gravity assist maneuvers and Trajectories G

In a gravity assist maneuver (or swing-by maneuver), a smaller body, such as a spacecraft, performs a close approach to a more massive body, such as a planet or a moon. This encounter, which changes the energy and angular momentum of the smaller body, has been used in several missions [2, 3, 4, 5] to alter the spacecraft velocity so it can reach far away objects without requiring extra impulsive maneuvers. The mathematical formulation for the energy gain due to the maneuver and further details about it can be found in Reference [6].

In turn, lunar swing-bys were also explored to accomplish several goals [7, 8, 9, 10]. In particular, starting from periodic orbits of family G [11], i.e. orbits around the Lagrangian equilibrium point L_1 in the Earth-Moon system (Figure 1), Reference [12] defined a set of trajectories, called Trajectories G of Escape (TGE, Figure 2), in which a spacecraft performs a lunar swing-by and escapes the system with low magnitude ΔVs.

Reference [12] presented results in which with a single ΔV, of approximate magnitude of 3.15 km/s, a spacecraft in a TGE is capable of reaching distances of 180.3×10^6 km (1.206 au) in the aphelion and 125.3×10^6 (0.8376 au) in the perihelion.

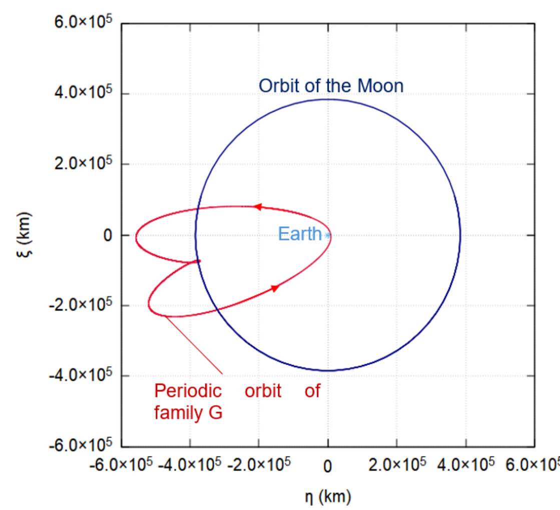

Figure 1: Periodic orbit of family G in the geocentric plane (η, ξ).

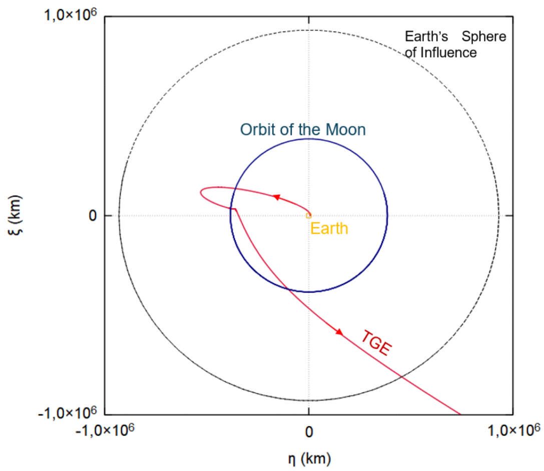

Figure 2: Periodic orbit of family G in the geocentric plane (η, ξ).

b) Genetic Algorithms

A Genetic Algorithm (GA) is a class of optimization algorithms that uses biological concepts as inspiration for their operation [13]. The GA works over a population of potential solutions for a given problem, in which each individual contains a set of characteristics, called genes, that are variables of this problem [14]. The GA simulates the evolution of the population using concepts of natural evolution as crossover and mutation to randomly create new individuals at each generation [13]. A fitness function, given by the mathematical formulation of the problem, is utilized to select the elements that are better suited to the environment, i.e., that optimize the solution of the problem [14].

GA were utilized in astrodynamics problems in various ways [15, 16], particularly to minimize the total increment of velocity of transfers, or the time of flight toward a target.

Methodology: This investigation is divided into two moments:

- The departure from Earth is analyzed under the dynamics of the Restricted Two- or Four-Body Problem, considering different escape trajectories, as introduced earlier. Numerical simulations are utilized to integrate the trajectories of the spacecraft, using a RADAU integrator of order 12 [17].
- As the spacecraft leaves Earth's Sphere of Influence (SOI), a GA is utilized to optimize its trajectory toward '2025PDC', using Lambert's Problem solution in the fitness function. At this point, the movement of the spacecraft toward the asteroid is analyzed solely under the Restricted Two-Body Problem.

a) System Dynamics

The differential equations of motion in the inertial heliocentric reference system (X, Y, Z) is presented in Eq. 1, in which $\mu = G \times m$ is the gravitational parameter, G is the gravitational constant, and m is the mass of each body, with index 1 corresponding to the Sun, 2 to the Earth, 3 to the Moon, and 4 to the spacecraft.

$$\ddot{\mathbf{R}}_i = \sum_{j=1 j\neq i}^{4} \frac{\mu_j}{R_{ji}^3}(\mathbf{R}_j - \mathbf{R}_i) \quad (1)$$

In Eq. 1, $\mathbf{R_i} = (X_i, Y_i, Z_i)$ is the vector position of the i-th body, while $R_{ij} = |\mathbf{R_j} - \mathbf{R_i}| = [(X_j - X_i)^2 + (Y_j - Y_i)^2 + (Z_j - Z_i)^2]^{\frac{1}{2}}$, with $j \neq i$, is the distance between the i-th and j-th bodies, and $\ddot{\mathbf{R}}_i$ is the acceleration of the i-th body.

Eq. 1 is utilized for both the simulations under the dynamics of the Restricted Two-Body Problem (R2BP) and Restricted Four-Body Problem (R4BP), with some remarks:

- The spacecraft is considered to have negligible mass, therefore, in Eq. 1, $\mu_4 \approx 0$, for all circumstances.
- For the R2BP Earth-Spacecraft, the spacecraft equation of motion becomes $\ddot{R}_4 = \frac{\mu_2}{R_{24}}(\mathbf{R_2} - \mathbf{R_4})$, i.e., $\mu_1 = \mu_3 = 0$.
- For the R2BP Sun-Spacecraft, the spacecraft equation of motion becomes $\ddot{R}_4 = \frac{\mu_1}{R_{14}}(\mathbf{R_1} - \mathbf{R_4})$, i.e., $\mu_2 = \mu_3 = 0$.

b) Hyperbolic Departure

The first step is to determine a launch date. As the discovery of "2025PDC" happens in 2024 and its probable impact with the Earth falls in April 2041, four scenarios are investigated, in which the launch date of the spacecraft happens up to 3, 7, 8 or 14 years before the estimated impact with the Earth.

The fitness function employed in the GA uses the Lambert's Problem to determine the ΔV necessary for the transfer. Lambert's Problem is a two-point boundary value in the R2BP [18], in which given an initial position vector, $\mathbf{R_i}$, a final position vector, $\mathbf{R_f}$, and the transfer time between these two positions, Δt, it is possible to define the velocity of the body at the initial and final positions, $\mathbf{V_1}$ and $\mathbf{V_2}$, respectively. Thus, a transfer trajectory between these two points is also defined. The problem is well-know and largely covered in reference textbooks of astrodynamics, such as References [19, 20].

The random variables of the Genetic Algorithm implemented are the launch date and the time of flight, (Δt), which are used to calculate: the initial position of Earth $(\mathbf{R_i})$, and the final position of the asteroid $(\mathbf{R_f})$. Then, $\mathbf{R_i}$ and $\mathbf{R_f}$, along with Δt, are used to calculate the velocity vectors $\mathbf{V_1}$ and $\mathbf{V_2}$.

Then, given $\mathbf{V_1}$, and using the patched conics approximation [19], it is possible to define the ΔV that will insert the spacecraft in a hyperbolic trajectory. For that, $\mathbf{V_\infty}$ is defined as the relative velocity between the spacecraft and the Earth, i.e, $\mathbf{V_\infty} = \mathbf{V_1} - \mathbf{V_{12}}$, in which $\mathbf{V_{12}}$ is the velocity of the Earth relative to the Sun.

Hence, using the Eq. 2, where V_c is the characteristic velocity of the circular parking orbit (for a 200 km altitude orbit, $V_c = 7.788$ km/s), V_p is the velocity of the spacecraft relative to the Earth at the periapsis of the hyperbolic orbit, r_p is the distance from Earth at this point, and and $V_\infty = |\mathbf{V_\infty}|$. Then, the ΔV is determined. The scheme for this approach is shown in Figure 3.

$$\Delta V = V_p - V_c = \sqrt{V_\infty^2 + \frac{2\mu_2}{r_p}} - V_c \qquad (2)$$

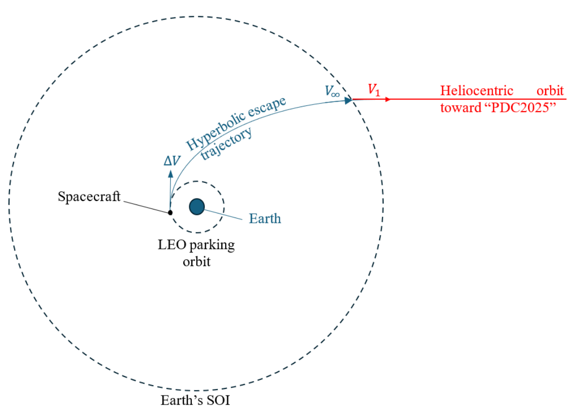

Figure 3: Scheme for hyperbolic departure.

For the optimizing process, the GA generates a initial random population. To create a next generation, the algorithm allows both the crossover of individuals and the generation of new ones. During this process there is a chance of a mutation occurring.

After that, there is a competition between the individuals, weighted by the fitness function. The ones that have better results survive and thus, a new generation is formed. The algorithm generates 50

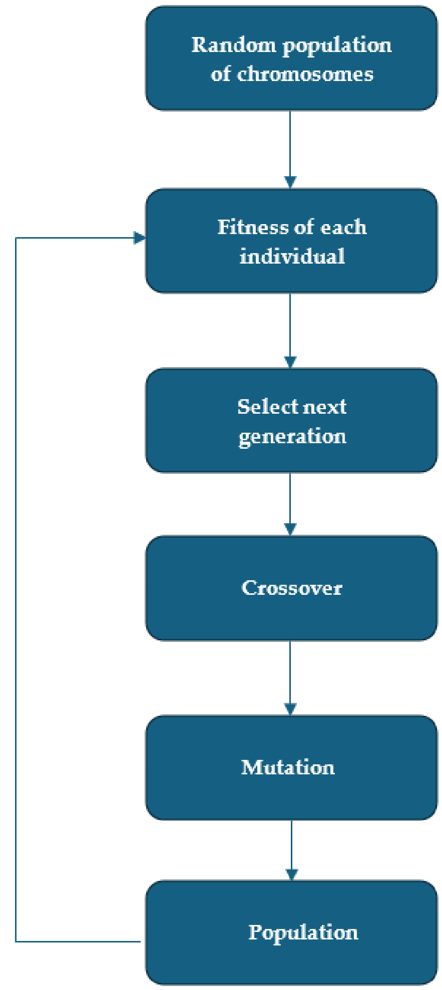

Figure 4: Genetic algorithm flow chart

generations, with 250 elements (Figure 4). Moreover, for this GA implementation, the Python library "pymoo" was utilized [21].

It's important to highlight that all this analysis is made under the R2BP. During the heliocentric transfer, is considered the R2BP Sun-Spacecraft, and from the LEO to the departure from Earth's SOI, the R2BP Earth-Spacecraft.

c) Lunar swing-by departure

Considering the launch dates for the hyperbolic departure, a time interval of 1 month that contains them was studied, such that the ephemerids of Earth and Moon were collected on the JPL Horizons Platform [22], with a step of 1 day. Using these values, and the dynamics of the R4BP, the simulation is started with the spacecraft in a circular LEO of 200 km of altitude around the Earth, the Moon in an elliptical orbit around the Earth and the Earth in an elliptical orbit around the Sun. The spacecraft is in the same plane of the Moon, relative to Earth's equator, and both start with the same true anomaly relative to the Earth, according with the values collected on the Horizon Platform.

A first increment of velocity, ΔV_1, is applied on the spacecraft in its direction of motion. This way, the spacecraft is injected in a TGE (similar to the one shown in Figure 2), with a characteristic velocity $V_0 = V_c + \Delta V_1$. Then, as the spacecraft performs a lunar swing-by, it leaves the Earth-Moon system, at a position $\mathbf{R_i}$.

At this point, the R2BP Sun-Spacecraft is once again considered and the GA is employed to define the Δt and the position $\mathbf{R_f}$ where the spacecraft will encounter the asteroid, with a fixed launch date, i.e, this part of the problem has only one random variable, the Δt. Thus, $\mathbf{V_1}$ and $\mathbf{V_2}$ are found.

Unlike what occurs for the hyperbolic departure, a second increment of velocity $\mathbf{\Delta V_2}$ is required to transfer the spacecraft from the TGE to the heliocentric transfer trajectory toward "PDC2025". Thus, the total ΔV for the transfer is the sum of the modules of $\mathbf{\Delta V_1}$ and $\mathbf{\Delta V_2}$.

d) Asteroid Deflection Evaluation

The NASA/JPL NEO Deflection App [23], in its "Intercept Mode", is employed to verify the efficiency in avoiding the impact of "PDC2025" with Earth. To utilize the application, the transfer time and the time of deflection (the number of days/years before the impact) are required. The former is one of the random variables of the GA, while the latter is determined by adding this amount to the launch date. Thus, both values will be know once the transfer trajectory is defined.

Results: This section is divided into two subsections, such that the transfer starting from the hyperbolic departure and from the TGE are presented separately.

a) Hyperbolic Departure

Table 1 presents the optimal results given by the GA for the trajectories that start from hyperbolic departures, in which the dates are presented in the "yyyy-mm-dd" format and V_{rel} is the module of the relative velocity vector between the spacecraft and the asteroid at the instant of impact. The highlights are the result for the 7-years Launch Scenario, in which the shortest transfer occurs, and for the 3-years Launch Scenario, which corresponds to the mission with the latest launch and also the smallest ΔV. These orbits are shown in the heliocentric plane in Figs. 5 and 6, respectively.

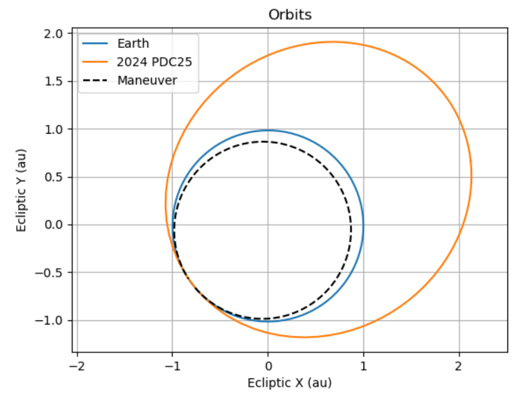

Figure 5: Trajectories in the heliocentric plane (3-years scenario - hyperbolic departure).

Table 1: Results for the hyperbolic departures

Launch Scenario	3 years	7 years	10 years	14 years
Launch Date	2038-04-26	2034-04-27	2031-04-27	2027-04-29
Interception Date	2039-03-11	2034-12-14	2032-10-31	2028-08-05
Time of Deflection (days)	775	2323	3097	4645
Transfer Time (days)	319	231	553	464
ΔV (km/s)	3.288	4.492	3.772	3.453
V_{rel} (km/s)	8.863	12.149	6.618	6.977

Utilizing the NASA/JPL NEO Deflection App with the data provided in Table 1, all transfers enable the deflection of the asteroid away from Earth. Although, for some, heavier interceptors were required. For example, for the 3-years Launch Scenario, the complete deflection is only possible for a delivered mass of 44.6×10^3 kg, launched by NASA SLS 2B. Figure 7 shows the orbit provided by the application, equivalent to the one shown in Figure 5, and the B-Plane, in which the deflection of the asteroid can be observed for all the launch vehicles. Table 2 shows the minimum mass of the in-

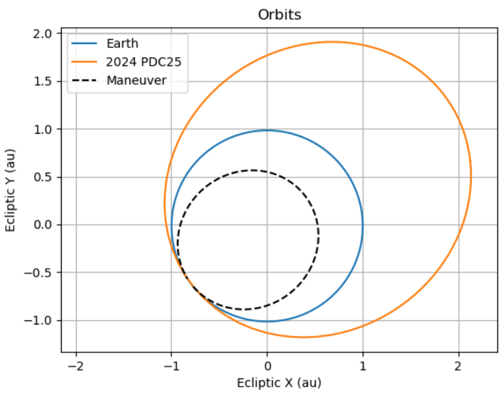

Figure 6: Trajectories in the heliocentric plane (7-years scenario - hyperbolic departure).

terceptor spacecraft that enables the deflection of the asteroid away from the Earth.

Table 2: Deflection by launch vehicles for the different launch scenarios (hyperbolic departure)

Launch Scenario	3 years	7 years	10 years	14 years
Launch Vehicle	NASA SLS 2B	Atlas V 551	Atlas V 551	Atlas V 551
Mass of the interceptor ($\times 10^3$ kg)	44.6	3.16	4.96	5.56

b) TGE departure

Similarly, Table 3 presents the optimal results for the trajectories that leave the Earth-Moon system via TGEs, noting that two ΔVs are required to the complete transfer, ΔV_1 to insert the spacecraft, and ΔV_2 to direct it toward "PDC2025". The total increment of velocity, ΔV_t is shown in Table 3. Furthermore, as the spacecraft takes approximately 20 to 40 days to perform the lunar swing-by and depart from the Earth-Moon system, it was added a "Departure Date", such that the time of deflection is calculated subtracting this date from the Interception Date.

Comparing Tables 1 and 3, it is possible to notice that the transfer times are similar, leading to even to same impact dates for some cases (3 and 10-years Launch Scenarios). Thus, the impact with the asteroid occurs in a similar way to the hyperbolic departures cases.

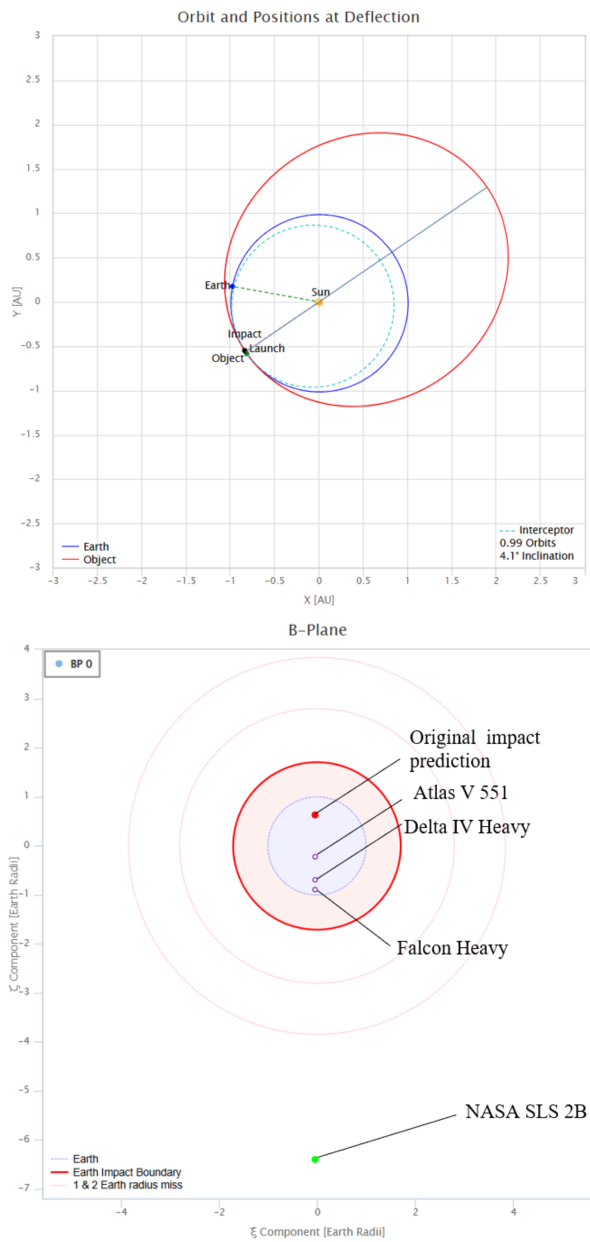

Figure 7: Interception for the 3-years Launch Scenario (hyperbolic departure)

However, the ΔV_t for the complete transfer are significantly larger than the single ΔVs required for the direct missions, with hyperbolic departures. These ΔV_t would make these missions unfeasible. Therefore, the deflection of the asteroid for these cases was not evaluated.

Conclusion: In this paper, a Genetic Algorithm was employed to investigate the optimum transfer, in terms of increments of velocity (ΔV), to asteroid "2024 PDC 2025", object of study of the 2025 PDC

Table 3: Results for TGEs departures

Launch Scenario	3 years	7 years	10 years	14 years
Launch Date	2038-04-16	2034-04-01	2031-04-16	2027-04-16
Departure Date	2038-05-22	2034-04-23	2031-05-23	2027-05-23
Interception Date	2039-03-11	2034-12-02	2032-10-31	2028-10-31
Time of Deflection (days)	775	2335	3097	4558
Transfer Time (days)	293	223	527	440
ΔV_1 (km/s)	3.159	3.160	3.161	3.160
ΔV_2 (km/s)	0.694	7.654	1.846	3.500
ΔV_t (km/s)	3.853	10.814	5.007	6.660
V_{rel} (km/s)	9.509	11.172	6.606	6.955

Hypothetical Asteroid Impact Scenario. Different scenarios were investigated regarding the departure trajectories from Earth and the launch and impact dates.

For hyperbolic departures, launch dates as late as 3 years before the impact with Earth were analyzed, and interception solutions with transfers time of approximately 300 days and $\Delta V = 3.29$ km/s were found, although, for the complete deflection of the asteroid, the launched mass is 10 times greater than the other scenarios. It is concluded that the 3-years scenario is critical, and would require great expenditures to make the mission possible. The 7-years scenario appears to be a good alternative, since it has the shortest transfer time and requires one of the smallest masses for the interceptor.

An alternative low-cost escape trajectory, in which the spacecraft perform a lunar swing-by was also considered, as the ΔV to eject the spacecraft from the Earth-Moon system (approximately 3.16 km/s) is even lower than the one to depart in a parabolic trajectory. However, since there is no control over the direction of the spacecraft's velocity vector when it reaches the limits of Earth's Sphere of Influence (SOI), the ΔV required to direct it to the asteroid becomes very large, making the mission unfeasible.

References: [1] K. W. Wierzchos, et al. (2024) *Minor Planet Electronic Circulars* 2024 ISSN 1523-6714. [2] K.-H. Glassmeier, et al. (2007) *Space Science Reviews* 128:1. [3] O. Grasset, et al. (2013) *Planetary and Space Science* 78:1. [4] C. Kohlhase, et al. (1977) *Space science reviews* 21(2):77. [5] H. F. Levison, et al. (2021) in *2021 IEEE Aerospace Conference (50100)* 1–10 IEEE. [6] R. Broucke (1988) in *Astrodynamics Conference* 4220. [7] J. Kawaguchi, et al. (2003) *Acta Astronautica* 52(2-6):189. [8] H. Chen (2023) *Astrodynamics* 7(3):315. [9] L. A. Gagg Filho, et al. (2018) *Computational and Applied Mathematics* 37:27. [10] J. L. Shannon, et al. (2021) in *31st Space Flight Mechanics Meeting*. [11] R. A. Broucke (1968) *Jet Propulsion Laboratory, Cal Tech*. [12] R. S. Ribeiro, et al. (2022) *Symmetry* 14(6):1132. [13] C. Toglia (2008) *Journal of Aerospace Engineering, Sciences and Applications* 1 `doi`. [14] D. Santos, et al. (2014) *Computational and Applied Mathematics* 34:1 `doi`. [15] N. A. Pallotta, et al. (2025) *Acta Astronautica* 228:285 ISSN 0094-5765 `doi`. [16] G. Neves, et al. (2019) *Journal of Physics Conference Series* 1365 `doi`. [17] E. Everhart (1985) in *Dynamics of Comets: Their Origin and Evolution* (Edited by A. Carusi, et al.) 185–202 Springer Netherlands ISBN 978-94-009-5400-7. [18] R. P. Russell (2019) *Celestial Mechanics and Dynamical Astronomy* 131(11):50. [19] R. R. Bate, et al. (2020) *Fundamentals of astrodynamics* Courier Dover Publications. [20] D. A. Vallado (2001) *Fundamentals of astrodynamics and applications* vol. 12 Springer Science & Business Media. [21] J. Blank, et al. (2020) *Ieee access* 8:89497. [22] J. D. Giorgini, et al. (2022) Nasa/jpl horizons on-line ephemeris system. [23] NASA/JPL (2025) NASA/JPL NEO Deflection App accessed on: March 29, 2025.