



2025 Revisions to the High Fluence NEO Deflection Formulae

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The planetary defense community practices mission-planning to deflect or disrupt any near-Earth asteroids (NEAs) on Earth-impacting trajectories. An initial analytic model to estimate the Δv imparted to a NEA by a stand-off nuclear explosive device (NED) was provided to NASA for this purpose ~ 10 years ago. In 2021 the model was revised and published¹. In 2023 a further revision^{2,3} was presented to better handle cases that had high X-ray fluences. This poster presents a third revision aimed at improved modeling of porous NEAs^{4,5,6,7} and adding more materials. The model is fit to hydrocode simulations initiated using Burkey et al.'s energy deposition model⁸. The uncertainty in Δv , when propagated from the uncertainty of the fit coefficients, shows the uncertainty increases greatly when the fluence is extrapolated.

Introduction

Mission planners need an analytic formula to estimate the Δv imparted to an asteroid by a standoff NED. This poster describes changes to the formula given by Managan & Burkey^{1,2,3} to include porosity and to add fits to a material model for Forsterite. To generate the data for the fits we initialized hydrocode runs using Burkey et al.'s energy deposition formula⁸. We ran a suite of simulations for 6 fluences, F , and 3-5 heights of burst, D , per fluence (see Figure 1). The suite was run for porosities, ϕ , of 10%, 30%, 50%, and 70%, and radii, R , of 100 and 250 m, and source temperatures of 1 and 2 keV for a total of 480 simulations per material. These simulations were run for the materials SiO_2 and Forsterite with no strength or damage model. Porosity is treated with a snowplow model. The Δv value converged at problem times of anywhere from 1 ms to 10 s.

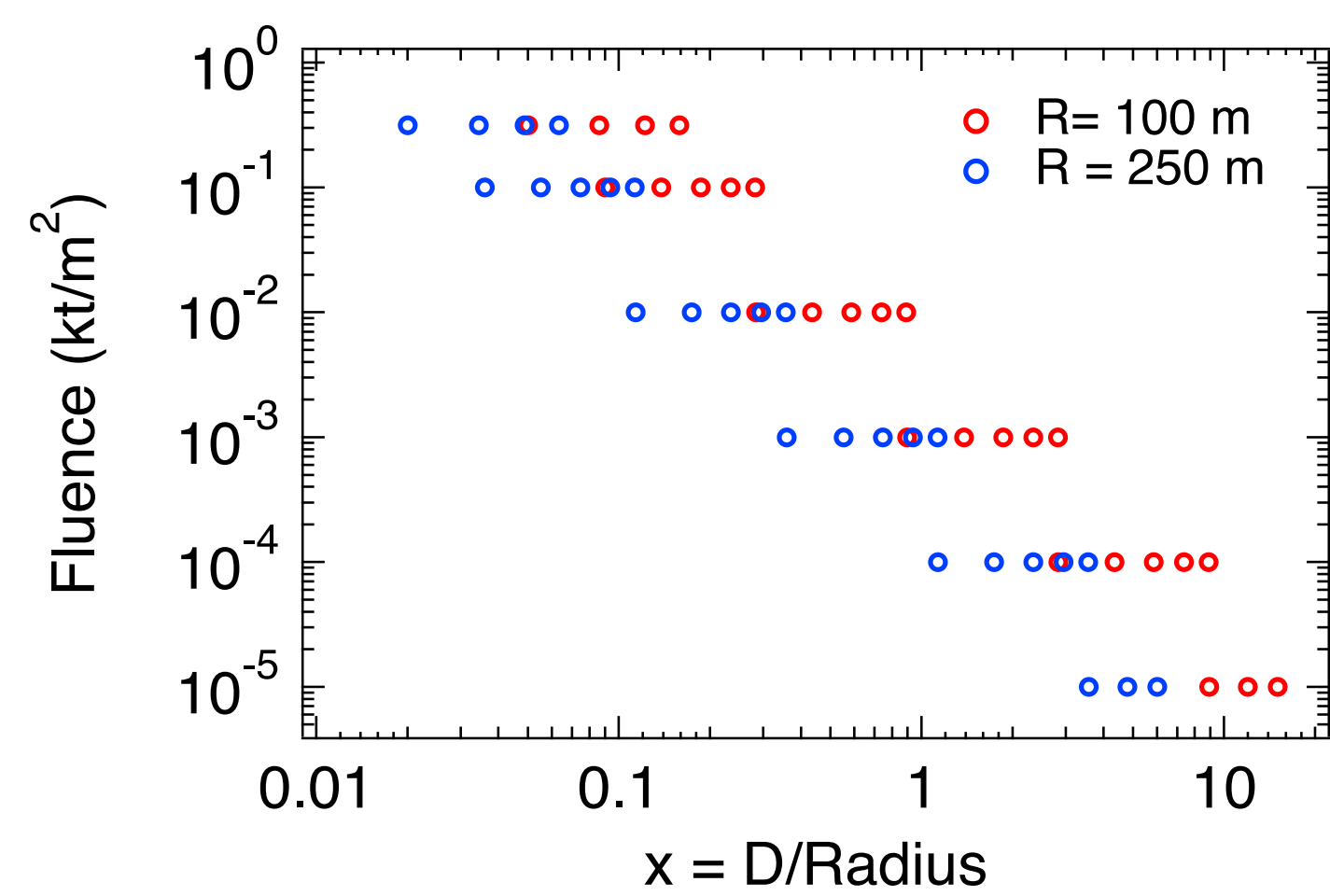


Figure 1. Fluences and scaled heights of bursts used in the simulations.

The Model

The Δv from the suite of simulations is used to fit the coefficients A and B from the analytic model. The model assumes an exponential deposition profile and calculates the melted mass using the B parameter. The model uses the variables, $x = D/R$, $y = Y/(B\rho D^2)$ where Y is the X-ray yield. The energy deposited is assumed to be completely turned into kinetic energy giving

$$\Delta v = \frac{A}{R^2} \sqrt{\frac{Y}{\rho}} \sqrt{\frac{x^2}{1+x} \left\{ \frac{2}{x} [1 + \ln y] - \left(1 + \frac{2}{x}\right) \ln \left(1 + \frac{2}{x}\right) \right\} \left[1 - \sqrt{1 - (1+x)^{-2}}\right]}$$

The first variant of this model¹, used 16 simulations by Joe Wasem and fitted constant values for A and B . In 2023 the model was revised to include simulations with higher fluences². To get an acceptable fit A and B were made quadratic functions of $\log(F)$, where F is the fluence in kt/m^2 .

In this poster we fit the model to Δv values calculated for multiple porosities. To get an adequate fit we found that A and B needed to depend on the porosity, ϕ , as well as $\log F$.

$$\ln A = (A_0 + A_3\phi) + (A_1 + A_4\phi) \log F + (A_2 + A_5\phi)(\log F)^2$$
$$\ln B = (B_0 + B_3\phi) + (B_1 + B_4\phi) \log F + (B_2 + B_5\phi)(\log F)^2$$

Dearborn's⁹ figure 11 showed that the response of an asteroid to a realistic NED spectrum is bounded by the response to a 1 and 2 keV spectrum. Therefore, we fit to the average of the 1 and 2 keV results in this poster. The fits to the individual temperatures are available in the GitHub repository³ mentioned in the Software section.

Dependence on ϕ is warranted

An F-test is used to confirm that the dependence of A and B on ϕ is warranted. When comparing fits with and without the porosity dependence the F statistics are over 100 and the associated p-values are effectively zero. Similar results were obtained when comparing fits quadratic in $\log F$ with only a linear dependence on $\log F$.

Therefore we let A and B have a quadratic dependence on $\log F$ and linear dependence on porosity. The question remains of how good the fit is. Figure 2 shows the relative residuals, $(\Delta v - \Delta v_{\text{fit}})/\Delta v_{\text{fit}}$, which have a FWHM of 12% and 18% for SiO_2 and Forsterite respectively. This level of agreement is considered adequate.

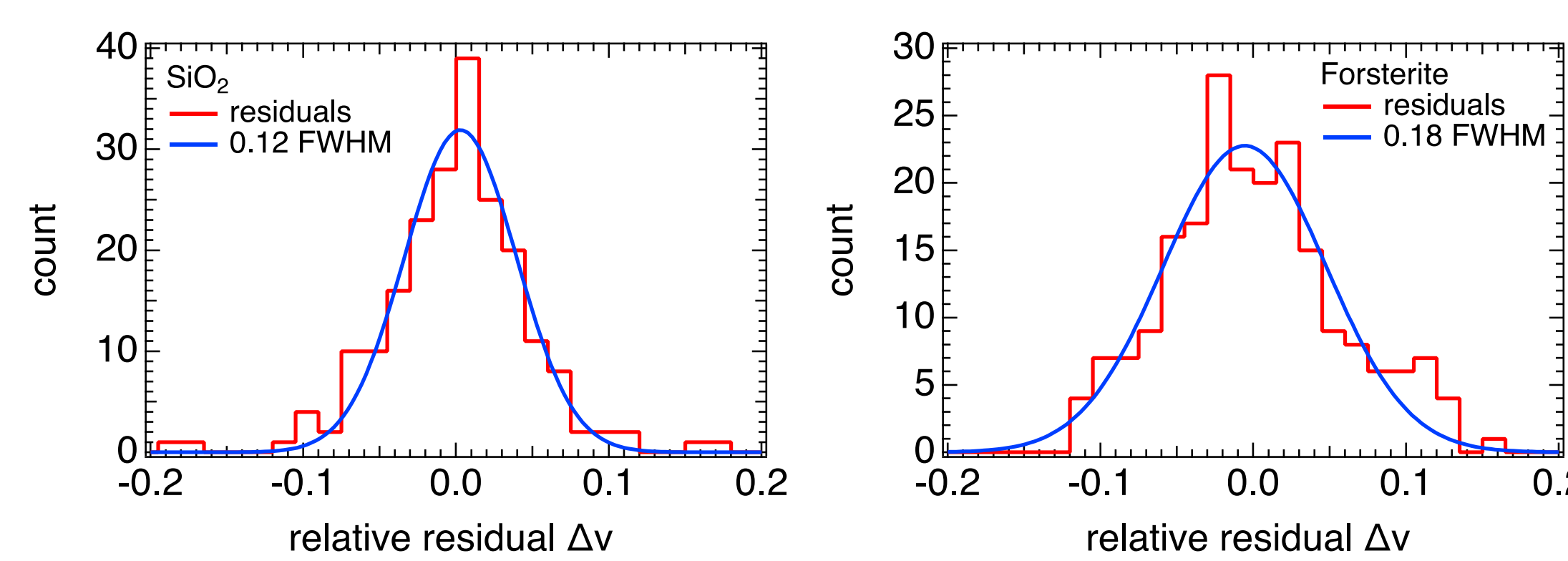


Figure 2. Relative residuals of the fit Δv , $(\Delta v - \Delta v_{\text{fit}})/\Delta v_{\text{fit}}$, and the simulated Δv for SiO_2 and Forsterite.

Statistical errors

The statistical errors for the twelve fitted coefficients of A and B were propagated to give the total uncertainty in Δv . The error from terms A_1 and A_2 grows with fluence and becomes a significant part of the overall error when the fluence is extrapolated above $0.3 \text{ kt}/\text{m}^2$, the largest fluence used in the fit. Figure 3 shows for the relative error in the fit values of Δv , i.e. $\text{error}/\Delta v$. The FWHM is small since no extrapolation is done here.

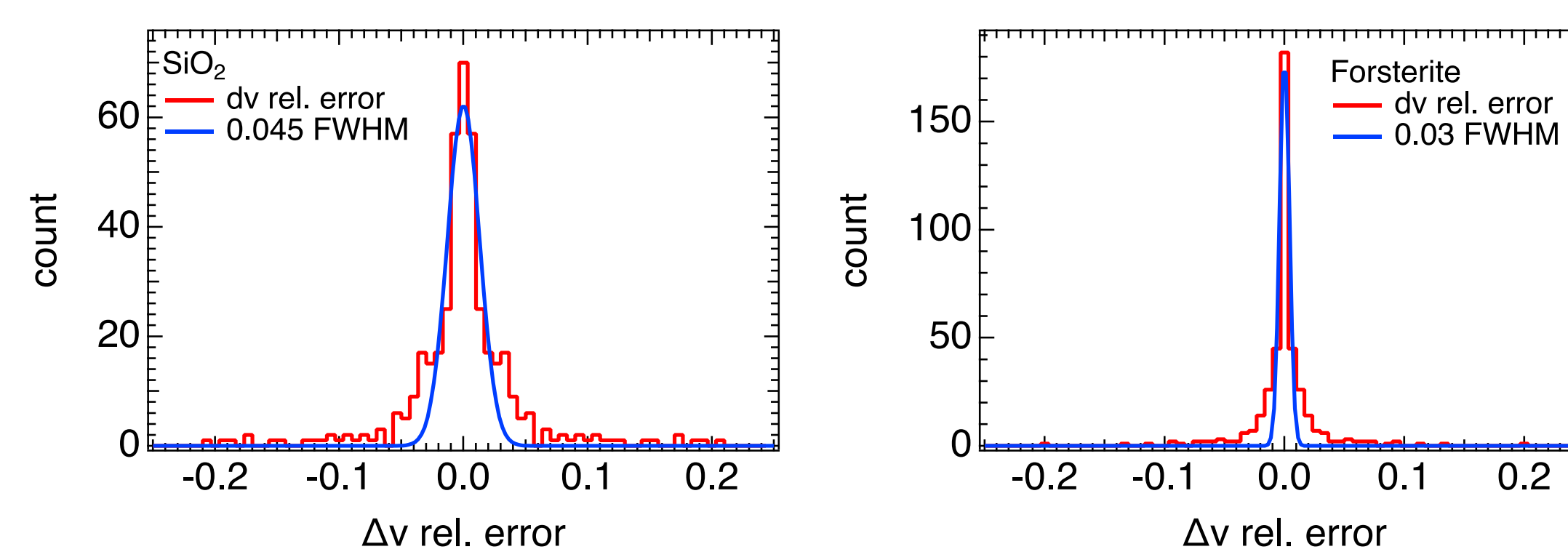


Figure 3. Relative errors of the fit Δv for SiO_2 and Forsterite.

Model errors

Our fit for Δv is done on the average of the 1 and 2 keV results as stated earlier. When fits are done separately to the 1 and 2 keV results the residuals for SiO_2 have a FWHM of 15% and 13% for 1 and 2 keV respectively. The Forsterite fits have a FWHM of 17% and 11%. These are comparable to the FWHMs for the fits to the average Δv given in Figure 2.

Figure 11 of Dearborn⁹ shows that the Δv for a realistic NED spectrum is different than the average of the Δv from the 1 and 2 keV simulations. This introduces a systematic error. A generous upper limit on the error is given by the difference of the fit from the Δv for the 1 & 2 keV simulations. That difference gives a relative error of about $\pm 45\%$. Therefore, the overall error should be larger than 12% and 18% for SiO_2 and Forsterite but well below 45%.

Practical use of the fits

One use of the fits is to find the optimal height of burst to get the largest Δv for a given yield. Figure 4 shows that the one sigma statistical error from the fit is not uniform but grows for small heights of burst as the fluence is extrapolated beyond the fitting data ($> 0.3 \text{ kt}/\text{m}^2$). The errors are large enough to make the optimal HOB uncertain.

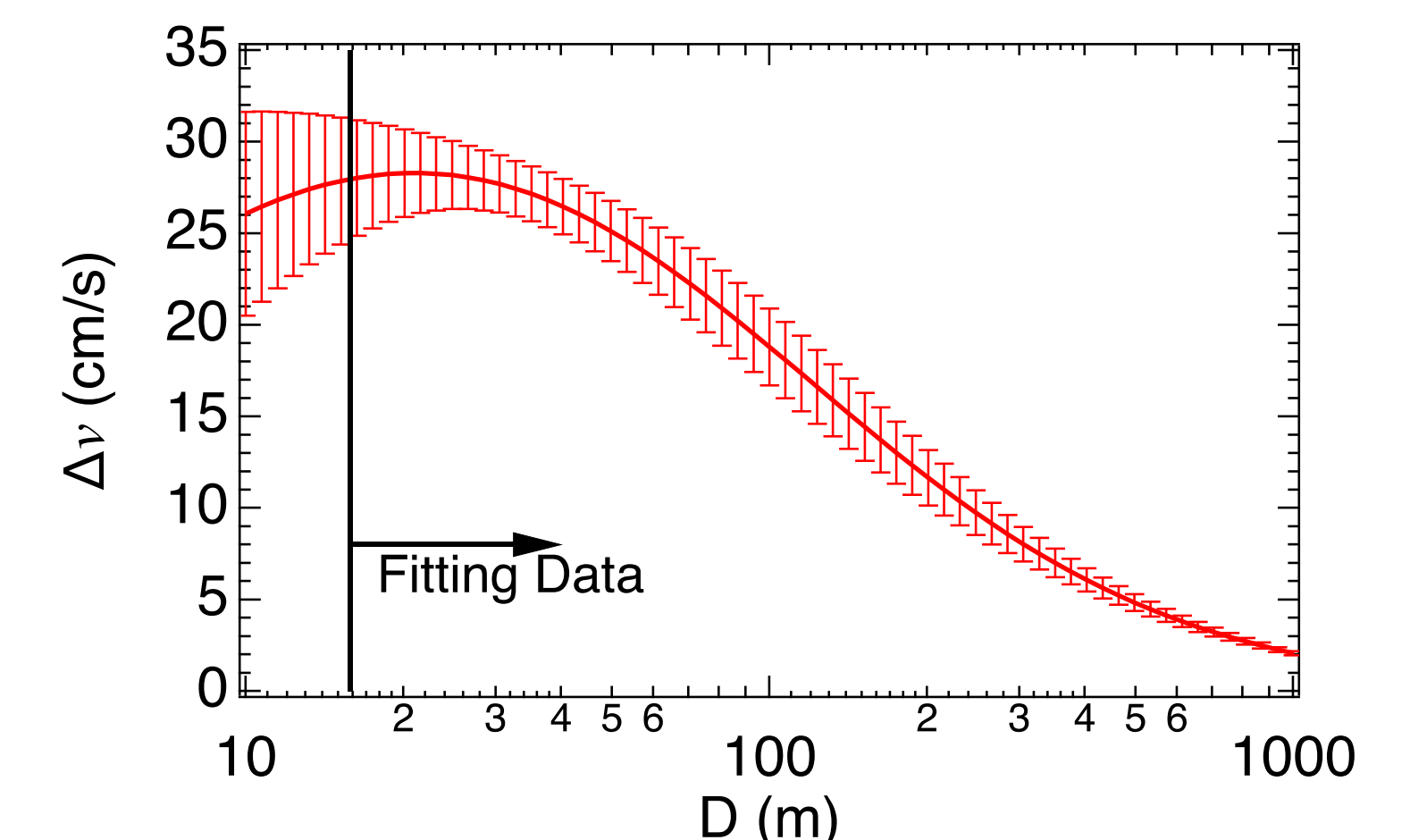


Figure 4. The Δv from the fit plotted with its statistical error. This is for SiO_2 with a yield of 1 Mt, a radius of 100 m, and $\phi = 0.245$

Software

The software to do the fits is available in a python module on a GitHub repository³. The 2nd release includes the updates described in this poster. It is located at https://github.com/LLNL/Analytic_Deflection_of_Asteroids_by_NEDs

A simple example is:

```
import numpy as np
import Deflection_formulae as DF

Yield = 1000.0
d = np.logspace(0., 3., num=100)
Rad = 100.
den = 2.00
por = 0.245
dv, dvError = DF.OriginalModel_Por_errs(d, Yield, Rad, den, por)
```

Conclusion

- The analytic model for predicting the Δv imparted by a NED has been refined to include the Forsterite material and the effects of porosity.
- The statistical error of the Δv is provided. The error can increase when the fluence is extrapolated beyond the fitted data. See Mary Burkey's poster IAA_PDC-25-06-37 for high fluence results for asteroid disruption.
- The fit residuals show a FWHM of 12% and 18% for SiO_2 and Forsterite respectively.
- An estimate of an upper limit on the error in Δv is given by looking at the spread between the 1 and 2 keV results for Δv . This upper limit is below 45%.

References

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