

## Abstract

The paper proposes an approach to designing transfer trajectories of a spacecraft from its initial orbit in the vicinity of the Sun-Earth libration point to near-Earth asteroids. The features of motion in bounded orbits around libration points, as well as invariant manifolds associated with them, open up the possibility of redirecting a spacecraft to trajectories of rendezvous with near-Earth asteroids almost without fuel consumption. The main focus of the research is to develop flight trajectories that involve a spacecraft initially approaching a celestial body and subsequently returning to the vicinity of the initial libration point.

The potentially hazardous asteroids Apophis and 2001 WN<sub>5</sub> were selected as target celestial bodies. The next close approaches of these asteroids to the Earth will take place in 2029 and 2028, respectively. It is worth noting that both near-Earth asteroids will first pass near the L<sub>2</sub> Sun-Earth libration point before approaching the Earth, and near the L<sub>1</sub> after the approach.

The James Webb Space Telescope, the Euclid spacecraft and the Spectrum-Roentgen-Gamma space observatory are spacecraft for which a concept of an asteroid exploration mission is proposed. All three spacecraft operate in orbits near the L<sub>2</sub> Sun-Earth libration point. As a result of the construction of invariant manifolds associated with halo orbits of these spacecraft, trajectories have been identified along which these spacecraft can approach the potentially hazardous asteroids Apophis and 2001 WN<sub>5</sub>. Based on the analysis of obtained trajectories, necessary impulses for the close passage to these celestial bodies were calculated. Trajectories leading to bounded orbits in the vicinity of the L<sub>2</sub> Sun-Earth libration point, after asteroids approaching, were also calculated. Preliminary results show that there are a number of possible scenarios for such flights, in which the total cost of the characteristic velocity does not exceed 50 m/s. It is shown that in all cases, all three spacecraft do not leave the area bounded by the so-called Kislik sphere of influence of the Earth – a sphere with a radius of 2.5 million km centered at the center of mass of the Earth.

The proposed concept may be useful for future missions at the libration points of the Sun-Earth system, especially for small spacecraft due to the low fuel costs for such flights.

**Keywords:** Sun-Earth libration points, near-Earth asteroids, invariant manifolds

## Introduction

The orbits of the James Webb Space Telescope [1], the Euclid observatory [2] and the Spectrum-Roentgen-Gamma space observatory [3] were chosen as the initial. These spacecraft are currently operating in the vicinity of the L<sub>2</sub> Sun-Earth libration point (see Fig. 1).

Asteroids Apophis and 2001 WN<sub>5</sub> [4] were selected as target celestial bodies.

The equations of a spacecraft motion in geocentric (index g) and heliocentric (index h) J2000 coordinate systems have the form:

$$\begin{cases} \ddot{\mathbf{r}}_g = -\frac{\mu_S}{r_{gS}^3} \mathbf{r}_{gS} + \mu_S \left( \frac{r_{gS} r_{gS}}{r_{gS}^3} - \frac{r_{gS}}{r_{gS}^3} \right) + \sum_{i=1}^n \mu_i \left( \frac{r_{gS} r_{gS}}{r_{gS}^3} - \frac{r_{gS}}{r_{gS}^3} \right), \\ \dot{\mathbf{r}}_g = \mathbf{V}_g, \\ \ddot{\mathbf{r}}_h = -\frac{\mu_S}{r_{hS}^3} \mathbf{r}_{hS} + \mu_E \left( \frac{r_{hE} r_{hE}}{r_{hE}^3} - \frac{r_{hE}}{r_{hE}^3} \right) + \sum_{i=1}^n \mu_i \left( \frac{r_{hE} r_{hE}}{r_{hE}^3} - \frac{r_{hE}}{r_{hE}^3} \right), \\ \dot{\mathbf{r}}_h = \mathbf{V}_h. \end{cases}$$

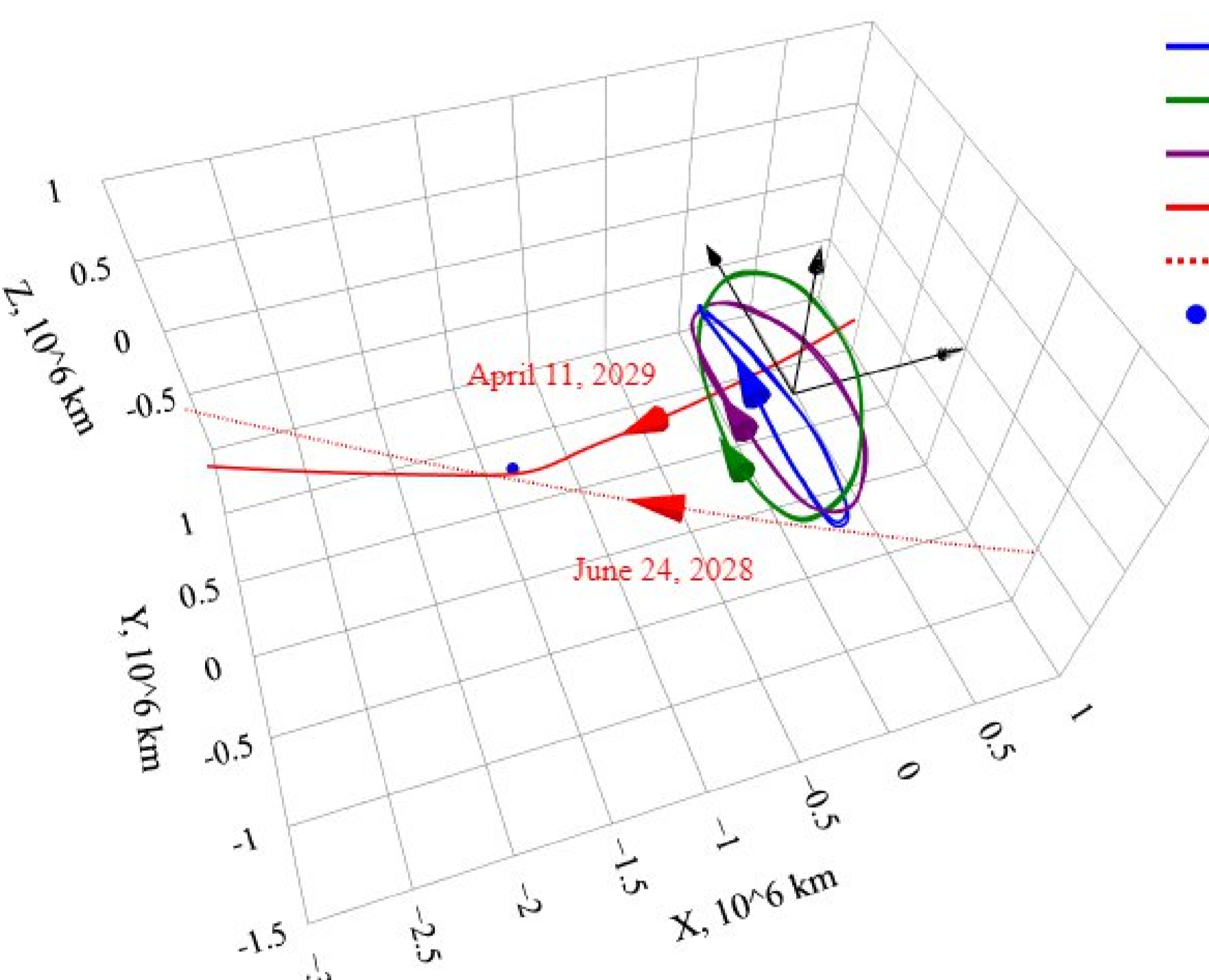


Fig. 1. Simulated JWST, Euclid, SRG, Apophis and 2001 WN<sub>5</sub> trajectories

The integration was carried out using the 8-order Runge-Kutta method in the JPL DE405 ephemeris model. All trajectories are represented in a rotating coordinate system centered at the L<sub>2</sub> Sun-Earth libration point

In this paper, we suggest a transfer scheme where a spacecraft, after applying  $\Delta V_1 = 1$  m/s along the escape direction [5], aligns with a trajectory from the unstable invariant manifold associated with its initial orbit near the libration point. For each pair of spacecraft-asteroid among the trajectories of the unstable manifold set, it is possible to determine one along which the spacecraft will pass at a minimum distance ( $\Delta r$ ) from the target asteroid.

Asteroid / Spacecraft	JWST	Euclid	SRG
Apophis	61900 km	33500 km	17600 km
2001 WN <sub>5</sub>	9900 km	94600 km	31800 km

The rendezvous of all three spacecraft with Apophis takes place on April 13, 2029. The encounter with the 2001 WN<sub>5</sub> asteroid for the JWST and SRG spacecraft takes place on June 27, 2028, while for the Euclid spacecraft - on June 26, 2028.

For the JWST-2001 WN<sub>5</sub> and SRG-Apophis cases, close approach trajectories ( $\Delta r < 1$  km) after applying  $\Delta V_2$ , as well as return trajectories (after applying  $\Delta V_3$  and  $\Delta V_4$ ) to the vicinities of the initial orbits, were calculated (see Fig. 3 and 6). All other schemes are shown in Fig. 2, 4, 5 and 7.

## JWST transfers

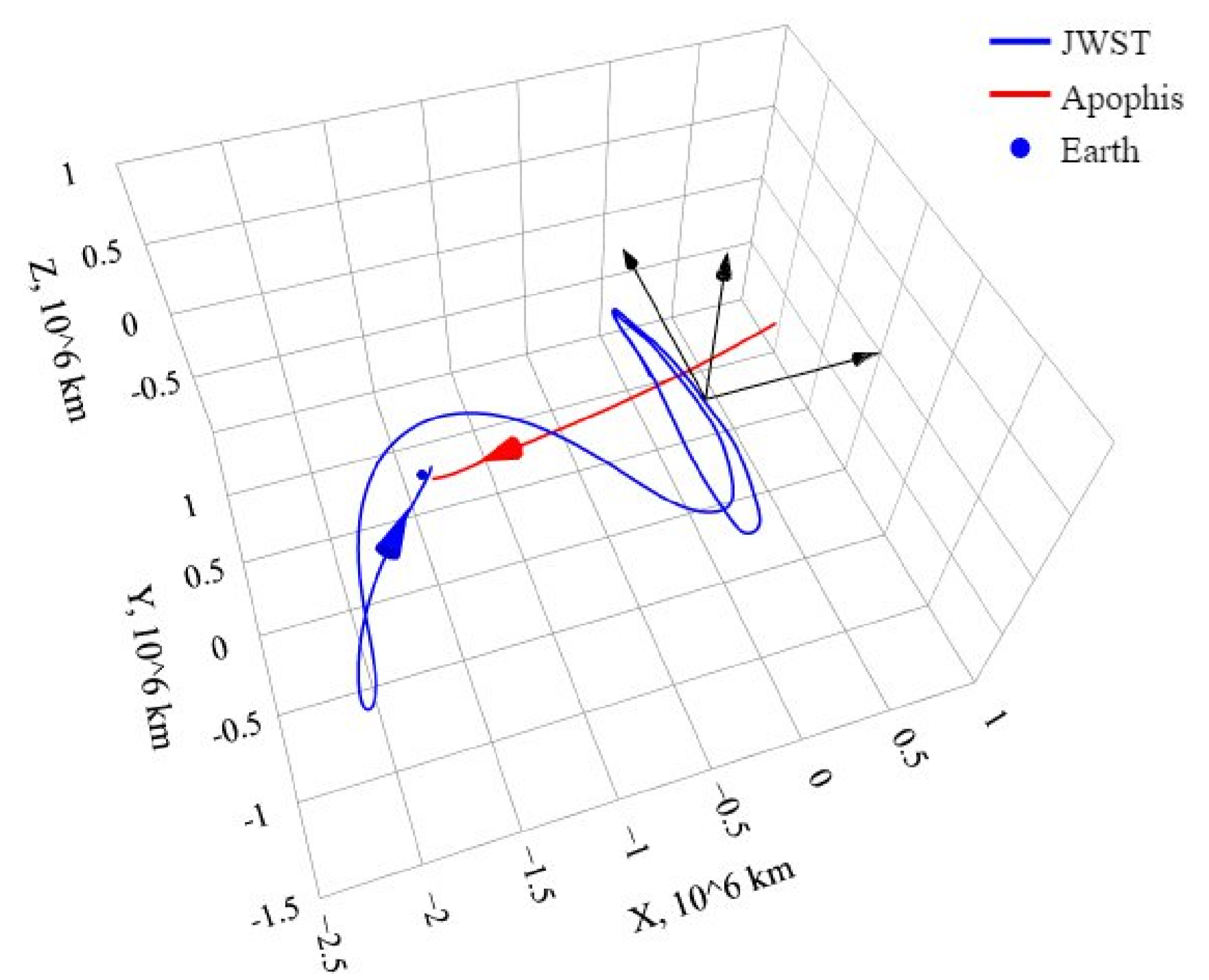


Fig. 2. Transfer trajectory of the JWST spacecraft to Apophis ( $\Delta V_1 = 1$  m/s,  $\Delta r = 61900$  km)

JWST-Apophis	
Impulse value	Date of the impulse implementation
$\Delta V_1$	September 2, 2028
JWST-2001 WN <sub>5</sub>	
Impulse value	Date of the impulse implementation
$\Delta V_1$	August 18, 2027
$\Delta V_2$	June 6, 2028
$\Delta V_3$	October 7, 2028

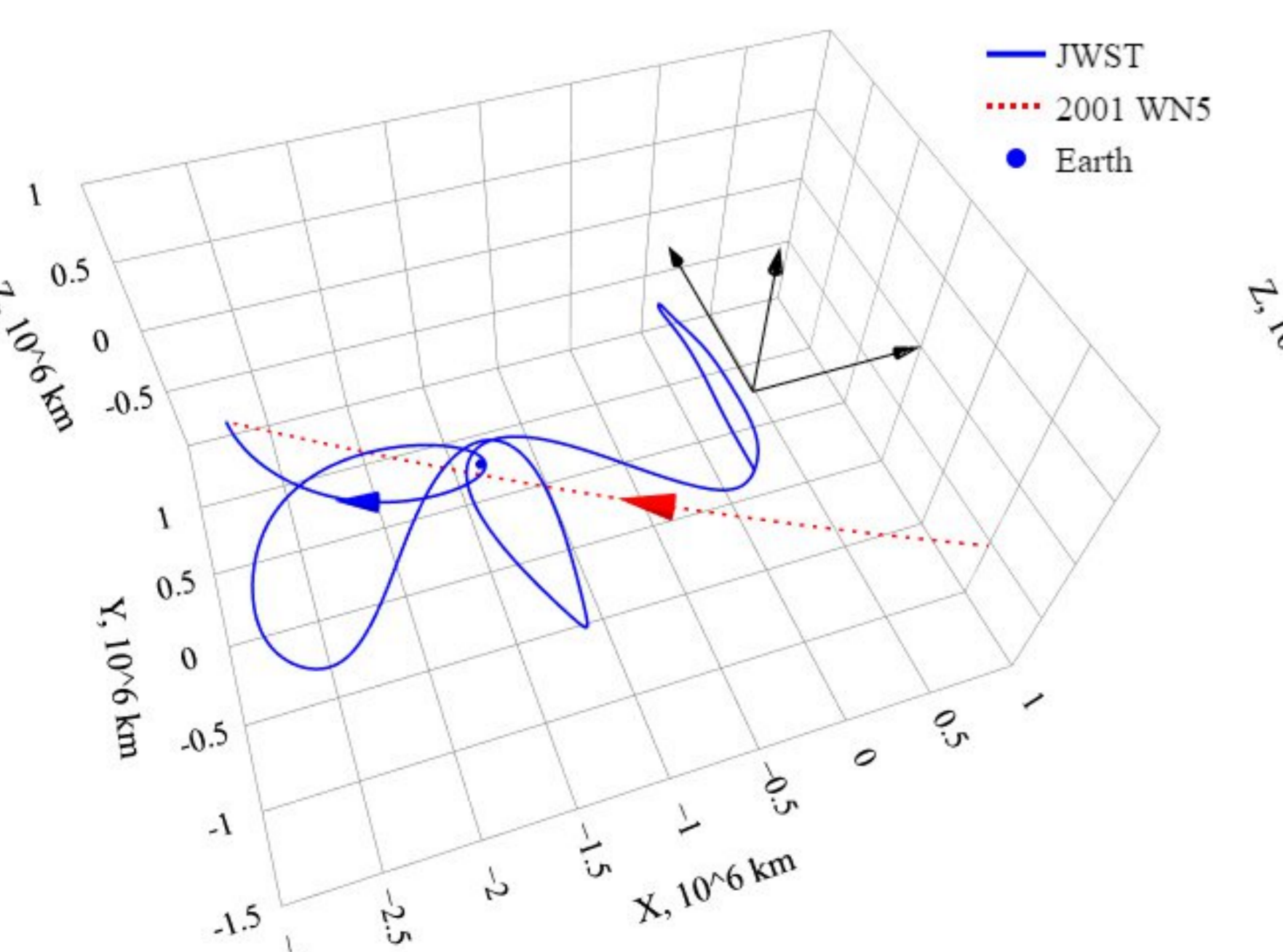


Fig. 3. Transfer trajectory of the JWST spacecraft to the 2001 WN<sub>5</sub> asteroid and return trajectory ( $\Delta V_1 = 1$  m/s,  $\Delta V_2 = 6.7$  m/s,  $\Delta r < 1$  km,  $\Delta V_3 = 2.2$  m/s)

## Euclid transfers

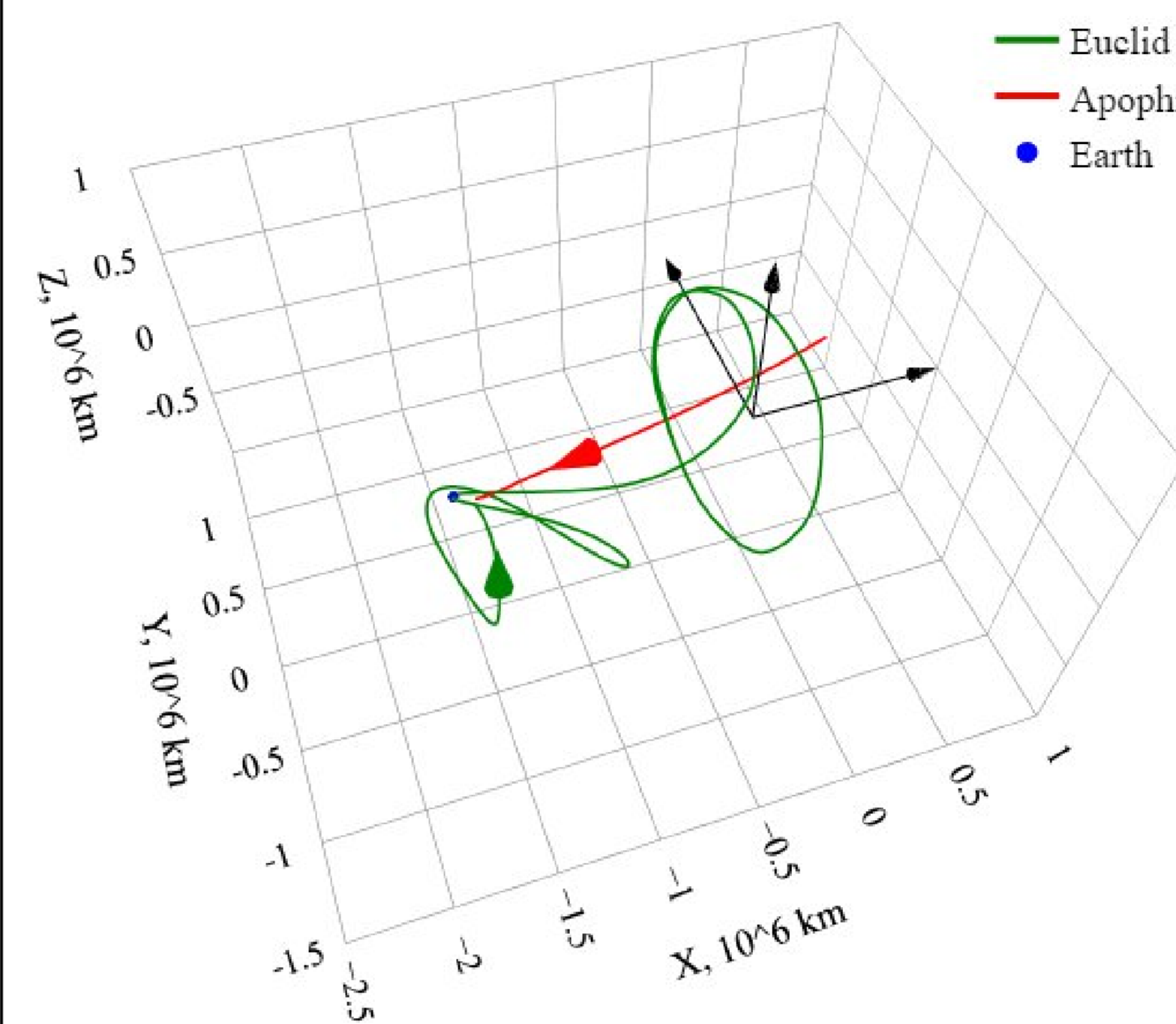


Fig. 4. Transfer trajectory of the Euclid spacecraft to Apophis ( $\Delta V_1 = 1$  m/s,  $\Delta r = 33500$  km)  
Date of the impulse implementation - August 19, 2028

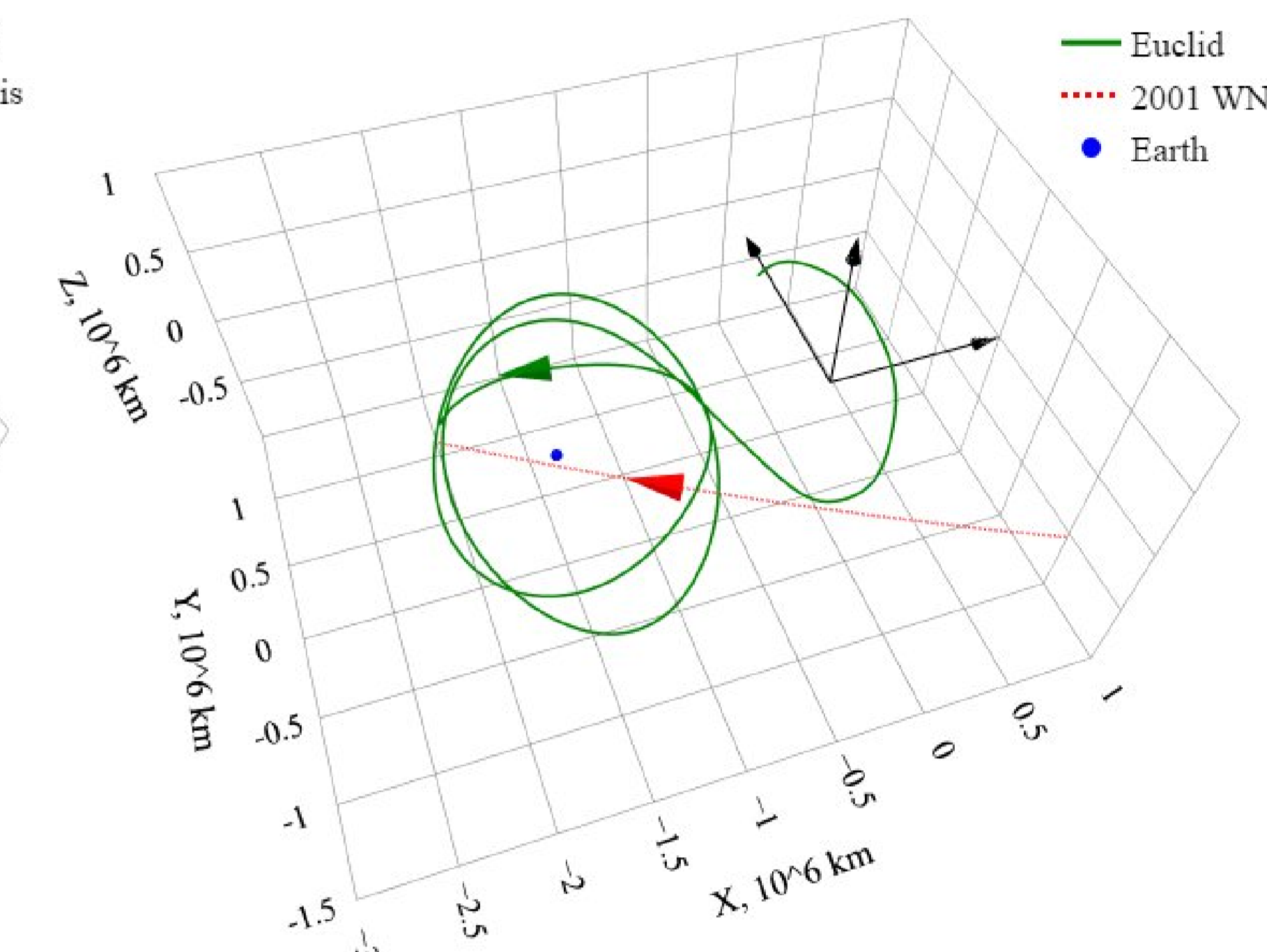


Fig. 5. Transfer trajectory of the Euclid spacecraft to the 2001 WN<sub>5</sub> asteroid ( $\Delta V_1 = 1$  m/s,  $\Delta r = 94600$  km)  
Date of the impulse implementation - May 16, 2027

## SRG transfers

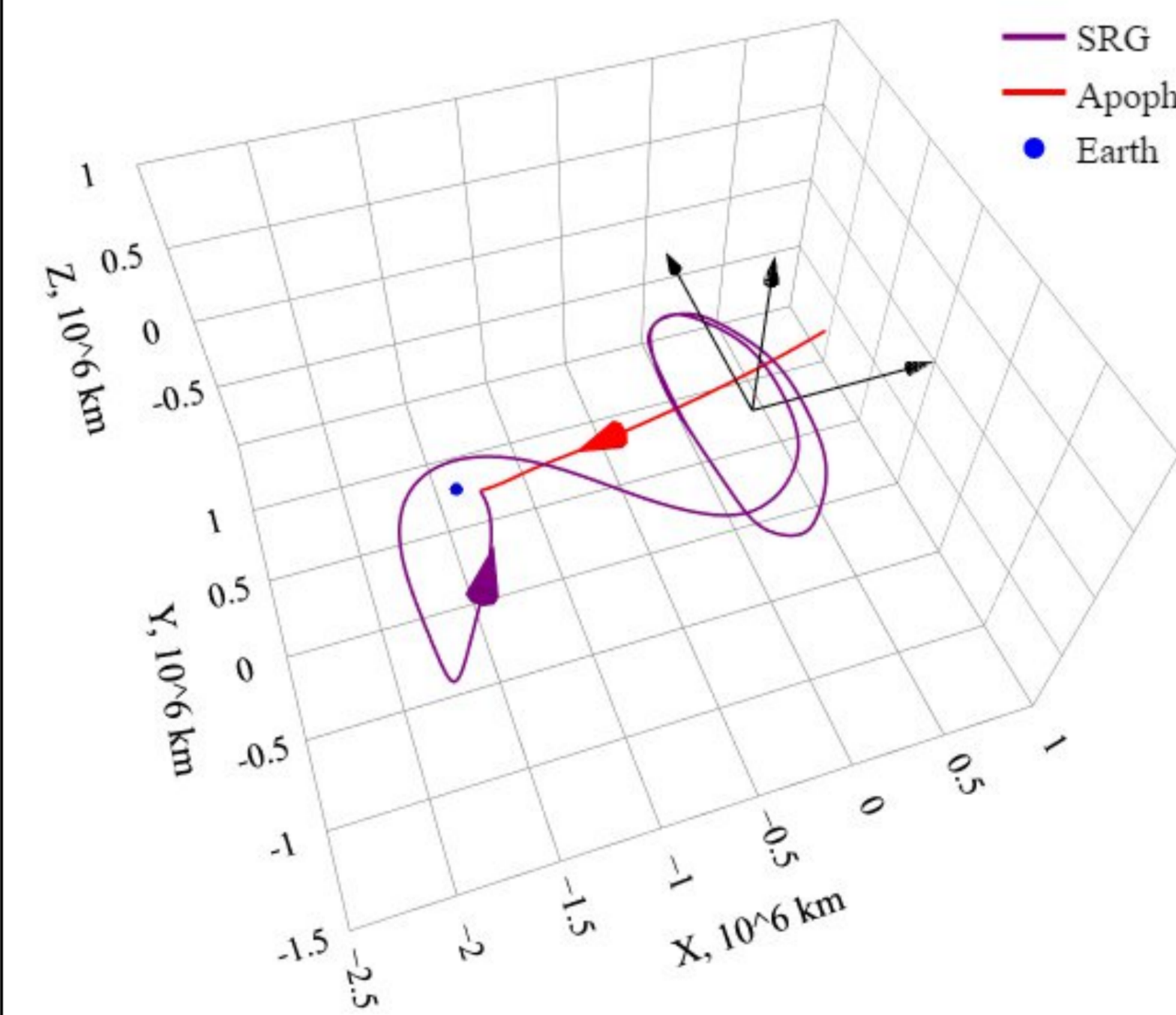
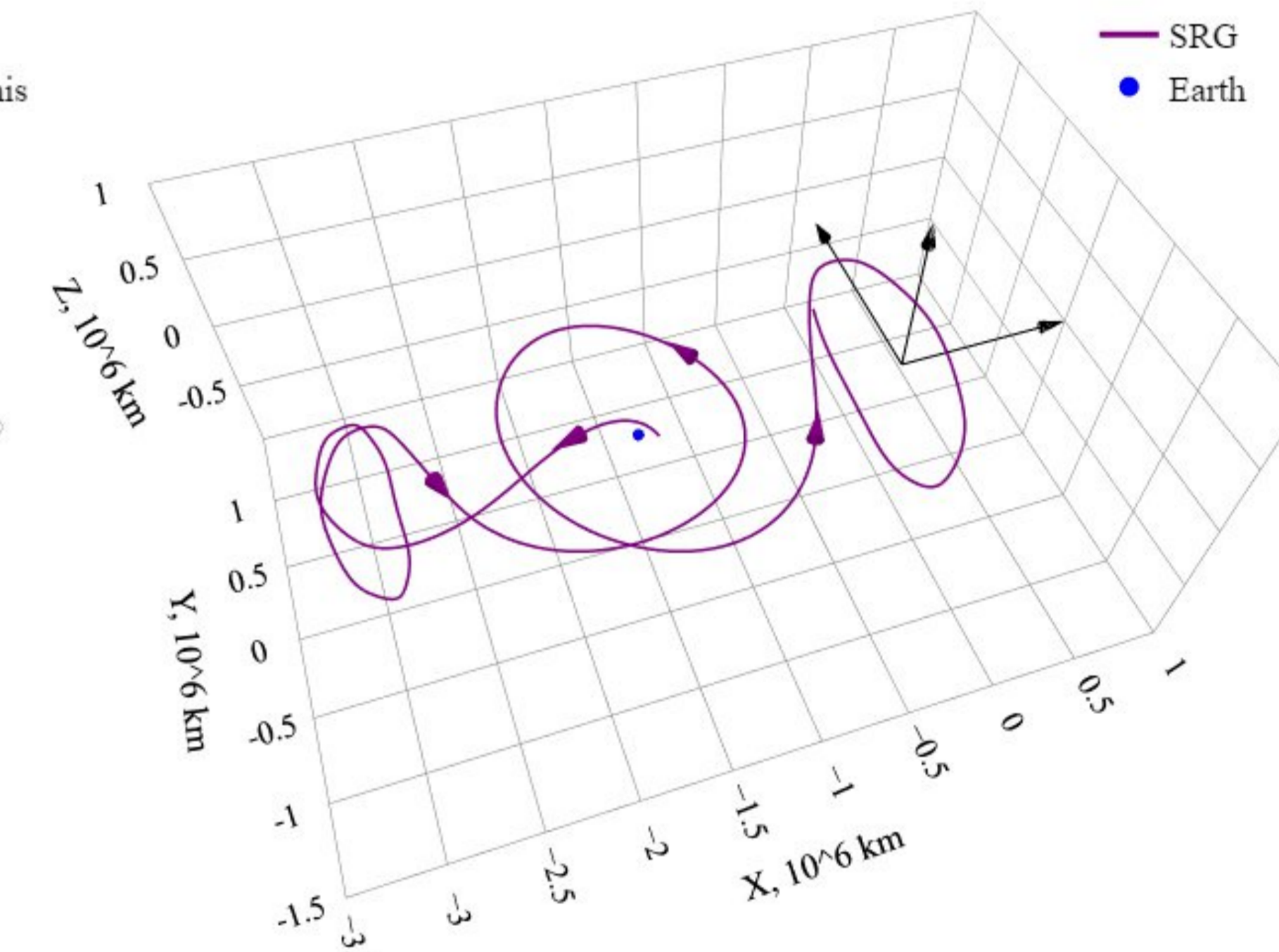


Fig. 6. Transfer trajectory of the SRG spacecraft to Apophis and return trajectory ( $\Delta V_1 = 1$  m/s,  $\Delta V_2 = 35$  m/s,  $\Delta r < 1$  km,  $\Delta V_3 = 6.2$  m/s,  $\Delta V_4 = 1$  m/s)



SRG-Apophis	
Impulse value	Date of the impulse implementation
$\Delta V_1$	September 13, 2028
$\Delta V_2$	January 17, 2029
$\Delta V_3$	April 14, 2029
$\Delta V_4$	October 17, 2029
SRG-2001 WN <sub>5</sub>	
Impulse value	Date of the impulse implementation
$\Delta V_1$	August 21, 2027

Fig. 7. Transfer trajectory of the SRG spacecraft to the 2001 WN<sub>5</sub> asteroid ( $\Delta V_1 = 1$  m/s,  $\Delta r = 31800$  km)

## Conclusion

This paper presents simulation results for spacecraft transfers to the Apophis and 2001 WN<sub>5</sub> asteroids using trajectories from invariant manifold sets. The study demonstrates that such transfers enable reaching the target celestial bodies with minimal propellant consumption. The proposed low-energy transfer concept could prove valuable for future missions to Sun-Earth libration points, particularly for small spacecraft where stringent fuel constraints apply.

## References

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