Impact probability predictions for 2024 PDC_{25} via jet transport techniques

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Introduction

Jet transport, also known as differential algebra, is a numerical technique based on automatic differentiation, used to calculate high-order expansions of analytic functions. It involves representing each quantity of interest as a truncated multivariate polynomial that depends on small deviations around a nominal value [5].

In this work, we apply jet transport techniques as implemented in the open-source software package `NEOs.jl` [3] to the problem of orbit determination and impact probability prediction for asteroid `2024 PDC`$_{25}$, the subject of the 2025 Hypothetical Asteroid Impact Scenario.

Orbit Determination

Given a set of $\{r_i\}_{i=1}^m$ observations, taken at times $\{t_i\}_{i=1}^m$ with observational uncertainties $\{\sigma_i\}_{i=1}^m$, the problem of orbit determination consists of minimizing the mean square residual function [2]

$$Q = \frac{1}{m}\sum_{i=1}^{m}\frac{\xi_i^2}{\sigma_i^2}, \tag{1}$$

where $\xi_i = r_i - R(\boldsymbol{x}(t_i), t_i)$ is the i-th observed minus computed residual, R is called the observation function and $\boldsymbol{x}$ is an orbit.

The standard method for optimizing the function Q, known as the differential corrections method, disregards the second derivatives of the residuals ξ to avoid increasing the computational load of the problem. In contrast, `NEOs.jl` uses jet transport techniques to compute a high-order polynomial expansion of Q, which is trivial to differentiate. Therefore, we are able to minimize Q via Newton method

$$\boldsymbol{x}_{k+1} = \boldsymbol{x}_k - \left(\frac{\partial^2 Q}{\partial \boldsymbol{x}^2}\right)^{-1}\frac{\partial Q}{\partial \boldsymbol{x}}. \tag{2}$$

In Fig. 1 we show the evolution of the Minor Planet Center uncertainty parameter for `2024 PDC`$_{25}$, as more observations become available. The initial orbit was computed using a jet transport version of Gauss method; then, orbits were refined using jet transport Newton method as explained above. For details see [4].

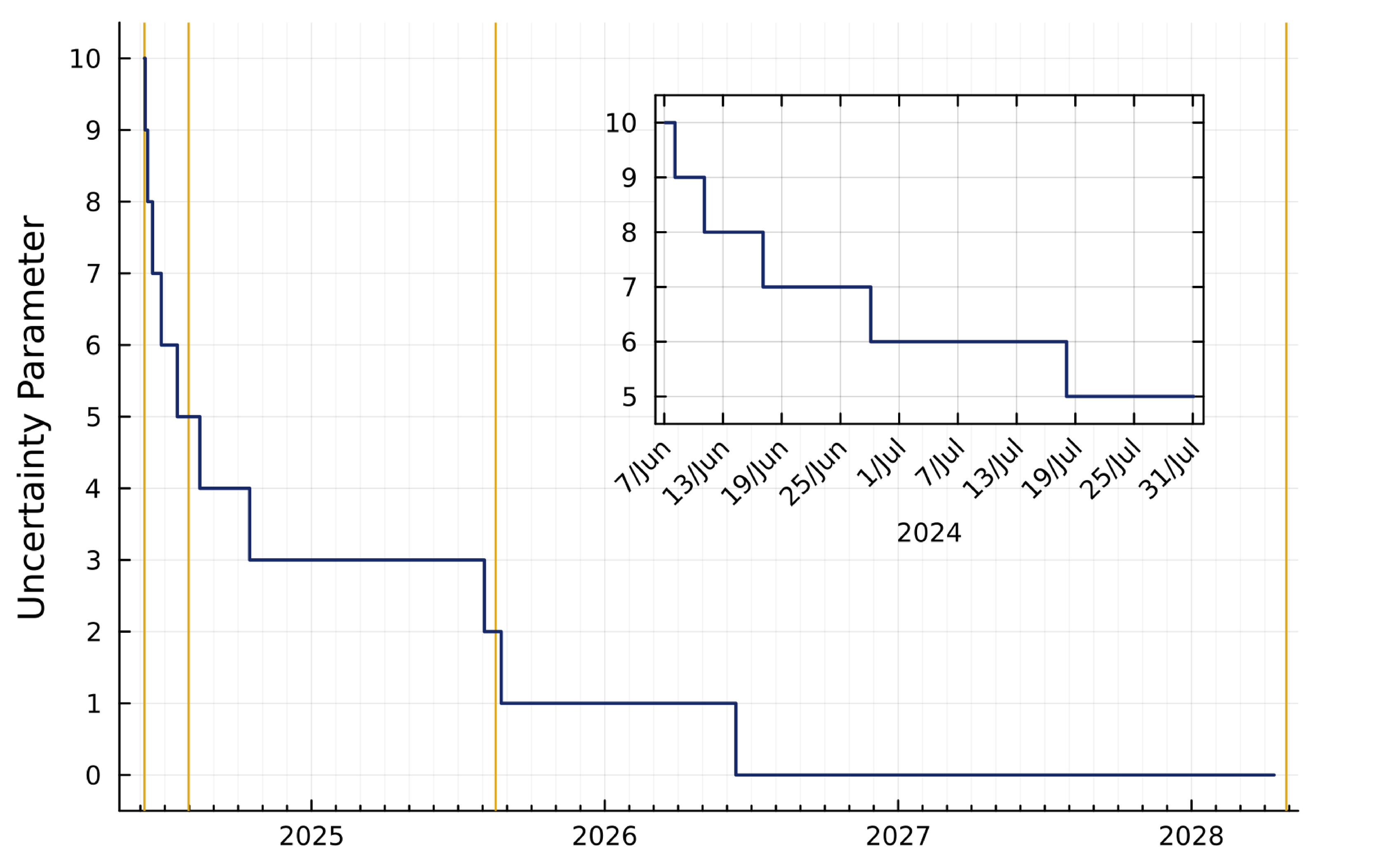

Fig. 1: Minor Planet Center uncertainty parameter as a function of the orbit computation date. From left to right, the vertical lines represent: the date of discovery, Epoch 1, the date the impact becomes certain (under the assumptions of this fictional scenario), and Epoch 2. The inset shows a zoom to the window between the date of discovery and Epoch 1.

Line of Variations

The line of variations (LOV) is a subset of the phase space, used for a fast estimation of the impact probability [2]. In `NEOs.jl`, we obtain a high-order expansion of the LOV by integrating the following differential equation with jet transport

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}\sigma} = k_1(\boldsymbol{x})\boldsymbol{v}_1(\boldsymbol{x}), \tag{3}$$

where $\boldsymbol{v}_1(\boldsymbol{x})$ is the eigenvector associated to the largest eigenvalue of the covariance matrix $\lambda_1(\boldsymbol{x})$, and $k_1(\boldsymbol{x}) = \sqrt{\lambda_1(\boldsymbol{x})}$. The solution of Eq. (3) is a curve parametrized by σ that follows the direction of largest uncertainty. To compute the Taylor series of the eigenvalues and eigenvectors of a matrix depending on the parameter σ, we use the algorithm by Mach & Freitag [1].

Given a nominal orbit $\boldsymbol{x}^*$, we can associate it with $\sigma = 0$ and sample the LOV with $2M+1$ points

$$-\sigma_{\max} \leq \sigma_{-M} < \sigma_{-M+1} < \cdots < \sigma_{M-1} < \sigma_M \leq \sigma_{\max}, \tag{4}$$

where $\sigma_{\max} = 5$. Each σ_i correspond to an orbit which can be propagated to study the close approaches of the object at hand. Now, we can substitute each σ_i by a jet transport polynomial $\sigma_i + d\sigma$, which will represent all orbits in the neighborhood of σ_i along the LOV. By propagating jet transport polynomials instead of points, we can obtain a piecewise high-order expansion of the LOV at any future epoch, which can be evaluated at any $\sigma \in [-\sigma_{\max}, \sigma_{\max}]$. Therefore, we do not need to propagate a large number of points to evaluate the LOV by linear interpolation. In other words, the main advantage of using jet transport for this application is substituting a systematic sampling and propagation of points on the LOV by polynomial evaluations.

In Fig. 2 we show the projection of the LOV over the 2041 target plane, computed using all the astrometry available at Epoch 1. The alternating colors visually distinguish the domain of each jet transport polynomial. Note that the density is higher near the middle, as points around the nominal orbit are more likely to represent the real trajectory. Also, note that the inset shows that the domain of two polynomials intersects the Earth's effective radius.

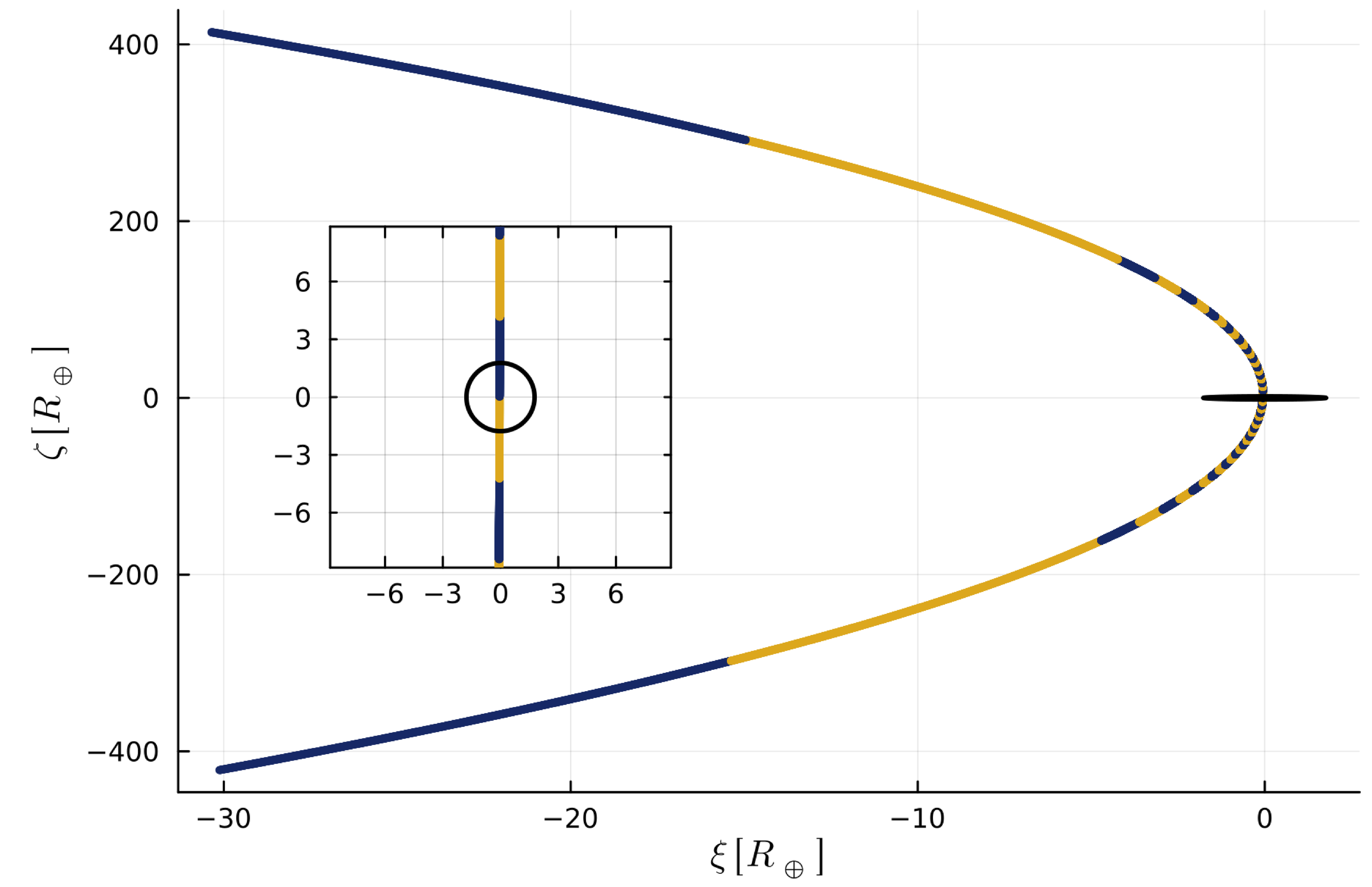

Fig. 2: Projection of the LOV over the 2041 target plane, using all observations available at Epoch 1. The alternating colors illustrate the domains of the jet transport expansions. The axes represent the Öpik coordinates in units of Earth radii. The inset shows a zoom around the Earth's circumference and the black circle corresponds to the Earth's radius enlarged by gravitational focusing.

Impact probability predictions

Given an analytical expression for the LOV, we can estimate the impact probability as

$$IP = \frac{1}{\sqrt{2\pi}}\sum_{i=1}^{n}\int_{a_i}^{b_i} e^{-\frac{\sigma^2}{2}}\, d\sigma, \tag{5}$$

where (a_i, b_i) is the domain where the projection of i-th jet transport polynomial crosses the Earth's effective radius. For instance, in Fig. 2, only two sub-domains have a nonzero impact probability. The number of polynomials we use to represent the LOV is given by $10U+1$, where U is the Minor Planet Center's uncertainty parameter.

In Figure 3 we show the evolution of the impact probability for `2024 PDC`$_{25}$, computed using Eq. (5), as more observations become available. We note a steady growth of the IP up to Epoch 1, with a sudden increase around August 2025, after which the impact becomes certain.

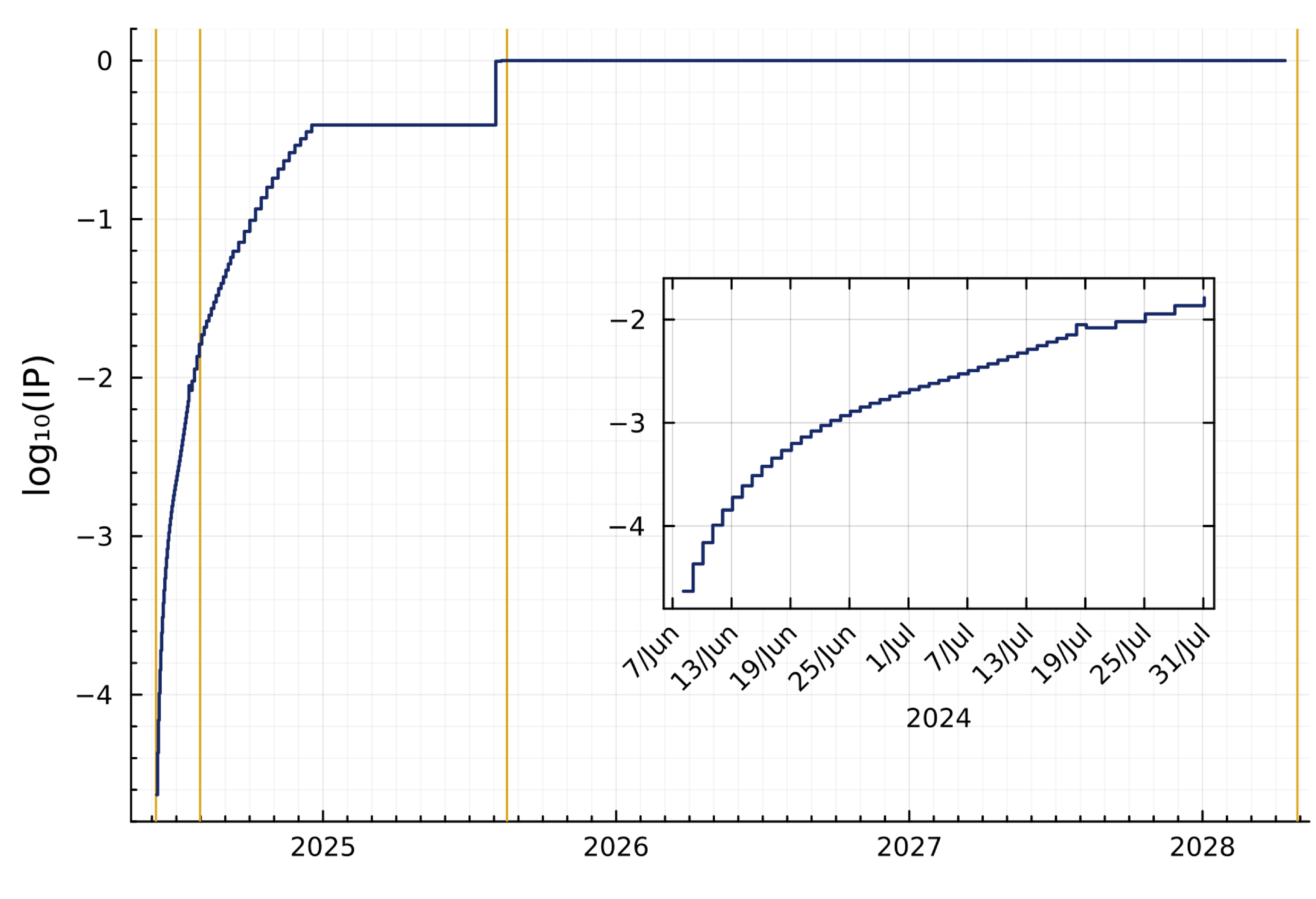

Fig. 3: Logarithm of the impact probability as a function of the orbit computation date. From left to right, the vertical lines represent: the date of discovery, Epoch 1, the date the impact becomes certain (under the assumptions of this fictional scenario), and Epoch 2. The inset shows a zoom to the window between the date of discovery and Epoch 1.

Acknowledgements

We acknowledge financial support from the PAPIIT-UNAM project IG-101122 and computer time provided through the project LANCAD-UNAM-DGTIC-284. L.E.R.M. acknowledges support from CONAHCYT.

References

[1] Thomas Mach and Melina A. Freitag. *Solving the Parametric Eigenvalue Problem by Taylor Series and Chebyshev Expansion.* 2023. arXiv: `2302.03661 [math.NA]`. URL: `https://arxiv.org/abs/2302.03661`.

[2] Andrea Milani and Giovanni Gronchi. *Theory of Orbit Determination.* Cambridge University Press, 2010.

[3] Jorge A. Pérez-Hernández, Luis E. Ramírez-Montoya, and Luis Benet. *NEOs.jl: v0.13.0.* Version 0.13.0. Apr. 2025. DOI: `10.5281/zenodo.14239533`. URL: `https://github.com/PerezHz/NEOs.jl`.

[4] Luis Eduardo Ramírez-Montoya, Jorge A. Pérez-Hernández, and Luis Benet. "Jet transport applications to the preliminary orbit determination problem". In: *Celestial Mechanics and Dynamical Astronomy* 137.3 (Apr. 2025), p. 16. ISSN: 1572-9478. DOI: `10.1007/s10569-025-10246-2`. URL: `https://doi.org/10.1007/s10569-025-10246-2`.

[5] Jianping Yuan et al. *Jet Transport Technique: Theory and Application.* Singapore: Springer Nature Singapore, 2024. ISBN: 9789819737215. DOI: `10.1007/978-981-97-3721-5`.